

HYDROCLIMATICALLY-DEFINED MIXED DISTRIBUTIONS IN PARTIAL DURATION FLOOD SERIES

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ABSTRACT. The possibility of mixed distributions in flood series is widely recognized and attempts to separate records into homogeneous subsets have been encouraged. Climate is often proposed as a source of mixed distributions, but the separation of flood records into climatic subgroups has tended to focus on seasonal divisions or a partitioning of the flood series into rainfall and snowmelt-generated events. In this study, a detailed hydroclimatic analysis is used to categorize floods on the basis of the synoptic weather patterns which produced them. This procedure represents the first attempt to classify events in a partial duration flood series according to the specific climatic flood-generating mechanism responsible for each event. The technique identifies mixed distributions by using physically-based information that is independent of the runoff series, rather than by defining subpopulations on the basis of the shape of the parent distribution itself.

INTRODUCTION

One of the most frequently cited areas of potential research in flood frequency analysis is the problem of mixed distributions or multiple populations in hydrologic time series. Homogeneity in the flood series is a basic underlying assumption for the probabilistic determination of flood magnitudes and frequencies. Wherever this assumption appears in the flood frequency literature, however, it is commonly followed by a disclaimer which states that due to differences in the climatic processes involved in the generation of floods, multiple populations or mixed distributions may be present in the data:

At some locations flooding is created by different types of events. For example, flooding in some watersheds is created by snowmelt, rainstorms, or by combinations of both snowmelt and rainstorms. Such a record may not be homogeneous and may require special treatment. (U.S. Water Resources Council, 1981, p. 7)

Despite the almost universal recognition that some observed flood samples may not be drawn from a single, climatically homogeneous population, only a handful of researchers have seriously devoted their efforts to the analysis of this problem. Potter (1958) was one of the first to discuss the evidence for two or more distinct populations of peak runoff (as seen in "dogleg" flood frequency curves), and he proposed possible climatic causes for the multiple populations. Singh (1968, 1974) presented a methodology for mathematically simulating mixed distributions in hydrologic samples, but although he referred to climate as a probable cause of multiple populations, his approach was to objectively search the streamflow data alone to define a mixture of distributions, rather than to decompose the data on the basis of additional climatic information. Other studies have attempted to identify mixed distributions in streamflow series by separating the flood record into seasonal subpopulations (Guillot 1973; Browzin et al. 1973). To date, the division of a record into seasons is probably the method most commonly used in attempts to identify the effect of climate on flood series homogeneity.

Jarrett and Costa (1982), Elliott et al. (1982), and Waylen and Woo (1982) moved beyond the simple seasonal division of a flood series and looked at the differences between rainfall and snowmelt-generated floods to examine the problem of mixed distributions in hydrologic data. By detailed examination of both streamflow and weather records, these researchers were able to subdivide flood series into rainfall and snowmelt "populations" so that separate flood frequency curves could be developed from each subset of data.

The separation of events in a flood series according to climate can be taken a step farther by identifying the various synoptic atmospheric circulation mechanisms and patterns that generate each flood event in a series. This is especially appropriate in climatically sensitive regions or in climatic transition zones where floods evolve from a variety of different processes that may be exhibited in complex frequency functions. This new hydroclimatic approach to defining mixed distributions in a flood series holds the promise of both enhancing our understanding of the flooding process and potentially improving flood frequency estimates.

THEORY OF MIXED DISTRIBUTIONS

The concept of a mixed distribution is based on the idea that a set of hydrologic observations, when sampled blindly from a population assumed to represent a single process or phenomenon, can also be interpreted as a composite sample representing several different processes or phenomena. In other words, the overall "parent" population may, in actuality, be composed of two or more subpopulations, each with its own distinct distribution. An example given frequently is that of annual floods arising from two sources, snowmelt and rainfall, each process producing floods having their own characteristic frequency distribution.

Figure 1 depicts a theoretical representation of the mixed distribution model. Events from two separate normal populations with different means and standard deviations may, when combined, form the top

distribution in the figure, which is the distribution that is "seen" or represented by the observed mean, standard deviation, and skewness of the data sample. In this example, the combination of the two non-skewed subpopulations results in a slightly skewed "mixed" population with a mean and standard deviation different from either of its two component parts.

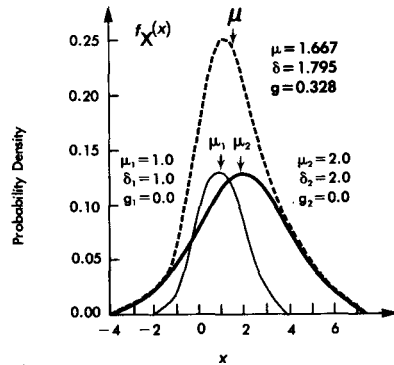


Figure 1. The mixed distribution model.

Different means (μ), standard deviations (σ), and skewness (g) are depicted for two component distributions, and the mixed distribution which they form (dashed line). (Modified from Hawkins, 1974)

Often, complicated frequency functions, such as those with multiple peaks or unusual skew, are more easily interpreted by thinking of the parent distribution as a mixed distribution which can be decomposed into several simpler frequency functions (Figure 2). In the simplest mixed distribution case, depicted in Figure 2a, each of the component distributions is considered to be normal with identical variances, hence the mixed distribution evolves only because of differences in the means of identically distributed frequency functions. An alternative model (Figure 2b) depicts the component distributions as having different means and variances, although still exhibiting normality. Frequency distributions of real-world phenomena need not necessarily decompose into simple normal frequency functions because nature does not always follow a Gaussian law precisely (Anderson 1967). A mixture of skewed and non-skewed distribution functions with various means and variances (Figure 2c) may be the model that best depicts the overall frequency distribution of a process arising from a multiplicity of causes, such as flooding.

The decomposition of hydrologic data into component distributions has significance for simulation studies, for extending frequency curves to predict rare events, and for increasing our understanding of the basic underlying phenomena in the flooding process (Hawkins, 1974). Studies which have concentrated on decomposing a flood series into component parts on the basis of the shape of the parent distribution alone have not always been able to address the underlying physical mechanisms responsible for each component in the distribution. An alternative approach to decomposition of the flood series is presented here. By classifying the events in a flood series according to the different types of climatic mechanisms that produce each flood, hydroclimatically-defined frequency components of the parent distribution can be isolated and examined. This separation of a flood frequency sample into a physically-based mixed distribution sample is a unique way to evaluate the importance of climate as a source of nonhomogeneity in a flood series.

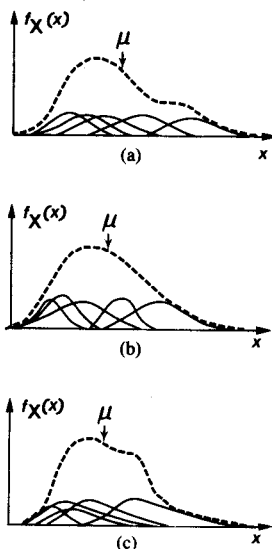


Figure 2. Complicated frequency functions decomposed into simpler components.

- (a) different means only
- (b) different means and variances
- (c) different means, variances, and skewness.

FLOOD HYDROCLIMATOLOGY IN THE GILA RIVER BASIN

In a hydroclimatic approach to flood analysis, hydrologic events are interpreted as occurring within the spatial and temporal framework of regional and global patterns of varying combinations of meteorological elements such as precipitation, storm tracks, fronts, air masses, and atmospheric circulation patterns. Flood hydroclimatology focuses specifically on the climatic genesis of different flood events.

The Gila River basin of central and southern Arizona (Figure 3) is an excellent study area for examining hydroclimatic aspects of the mixed distribution problem because the location, physiography, and climate of this area allow a variety of physical processes to produce flooding in different parts of the basin and at different times during the year. Most of the western part of the basin lies within the Sonoran Desert and is characterized by extremely arid conditions, sparsely vegetated flat-floored alluvial valleys, and ephemeral stream systems which flow largely in response to local convective showers. The Gulf of California and the Pacific Ocean are close enough, however, to provide a source for large influxes of moisture which are occasionally steered into the region either by tropical storms or other kinds of atmospheric circulation patterns. In the range and basin topography of the central and southeastern portions of the basin, slightly higher elevations and frequent rainfall due to orographic effects result in increased intermittent streamflow. At times, exceptionally large flows and flash floods are possible when excess moisture enters the region from either the Gulf of Mexico, humid regions of Mexico, the Gulf of California, or the Pacific Ocean. In the northern and northeastern parts of the basin, many streams draining the high elevations of the Mogollon Rim are able to maintain perennial flow due to an enhanced orographic effect and the accumulation

of sufficient snowfall during winter storms to contribute to spring snowmelt flooding. Local convective thunderstorms are a common source of flash flooding throughout the basin, especially during the humid summer months, and large frontal storms frequently sweep through the basin in winter.



Figure 3. The Gila River basin. Numbered symbols show locations of gaging stations. (A complete description of the stations is given in Hirschboeck, 1985.)

An example of an actual hydroclimatic sequence that resulted in widespread flooding throughout the Gila River basin is depicted in Figure 4. During September 4-6, 1970, several major synoptic features operated together and introduced large amounts of precipitable water vapor into the region, as well as a triggering mechanism to release this moisture. In the upper air circulation, a deep trough of low pressure steered moisture from the Pacific Ocean into southwestern United States, and on September 5th and 6th, the trough began to form a cutoff low circulation, causing the moisture and storminess to persist in the region instead of being moved along rapidly to the east in the upper air westerly flow. At the same time, Tropical Storm Norma tracked very close to the Baja California coast and introduced additional large fluxes of moisture into Arizona at both surface and above-surface levels in the atmosphere. Also concurrently, a surface cold front associated with the upper air trough, moved rapidly across the Gila River basin and acted as a lifting mechanism for the large amounts of moist, unstable air in the region.

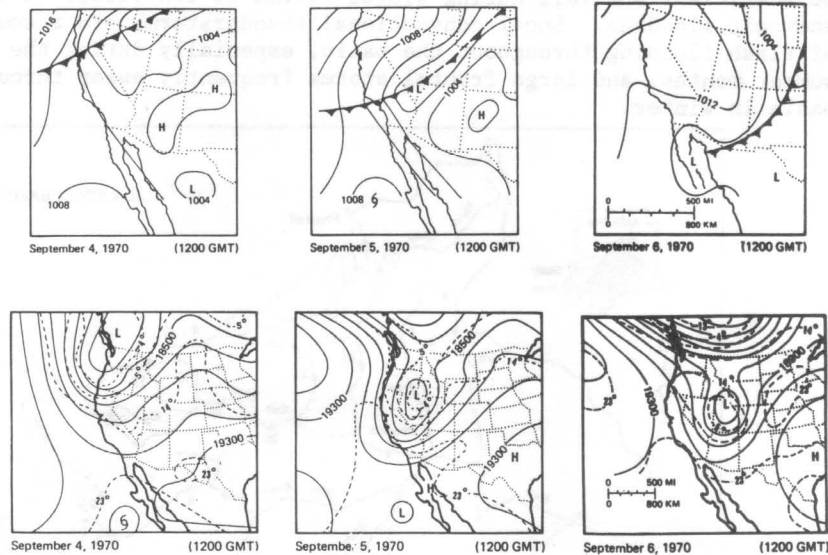


Figure 4. Hydroclimatic sequence which produced widespread flooding in the Gila River basin. (Top) surface weather maps for September 4-6, 1970. (Bottom) Corresponding 500 millibar charts. (Source: Hansen and Schwarz, 1981)

The above combination of synoptic-scale events generated flooding in the basin in a different way than the flooding that typically arises from local summer convective storms, isolated winter frontal passages, or spring snowmelt. The relevant question for the mixed distribution problem is whether or not these differing flood-generating mechanisms each produce their own unique frequency distribution of flood magnitudes. The arid and semiarid streams of the Gila River basin are especially well-suited to explore this question because sensitivity to climatic variability is high in ephemeral and intermittent streams.

Runoff hydrographs of streamflow in the Gila River basin can be strikingly different at different times of the year, due, in part, to the kinds of atmospheric generating mechanisms which produce the flows. For example, a flow event occurring in response to an intense summer convective thunderstorm might exhibit a hydrograph similar to Figure 5a, while a flow event produced by a winter frontal storm would show a longer rise time, less of a peak, and a longer recession time (Figure 5b). Floods produced by rare extreme events, such as tropical storms, cutoff lows, or a combination of processes, exhibit complex hydrographs (Figure 5c) which reflect the great magnitude of moisture delivered to the watershed, the complex nature of the storm itself, and the complicated nature of the catchment's response to a synoptic-scale delivery system.

These differences in the flood discharge hydrographs generated by different types of atmospheric processes in the Gila River basin suggest that the frequency distributions of the discharges may also vary under different types of flood-generating atmospheric processes and hence be a major cause of mixed distributions in observed flood series.

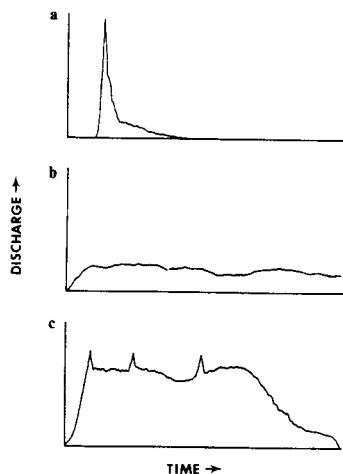


Figure 5. Idealized Gila River basin hydrographs resulting from different atmospheric mechanisms.

- (a) summer convectonal shower
 - (b) winter frontal storm
 - (c) tropical storm/cutoff low combination.
- (Modified from Keith, 1981)

DATA AND METHODS

In order to address the question of whether different hydroclimatic flood-generating mechanisms produce their own unique frequency distributions, flood series from thirty U.S. Geological Survey gaging stations in the Gila River basin were analyzed (Figure 3). The stations were selected to reflect a wide range of conditions in the basin and to provide an adequate sample with which to examine multiple causes of flooding. To avoid the influence of nonclimatic sources of nonhomogeneity, care was taken to select watersheds with as few diversions and upstream impoundments as possible, although nearly all streams but the very smallest drainages along the Mogollon Rim were used as sources of irrigation or municipal water supplies to varying degrees. No major reservoirs were located upstream from any of the selected stations.

The flood events studied were those of the U. S. Geological Survey's "peaks-above-base" or partial duration series data for the period 1950 to 1980. Partial duration series data were used so that a large spectrum of floods could be analyzed to better define the shapes of the distributions of various flood-generating mechanisms. In order to compare the relative magnitudes of peak flows from station to station, the flood values were transformed to base ten logarithms and standardized to dimensionless z-scores. After assigning a hydroclimatic classification to each flood event, histograms of the hydroclimatic subgroups were plotted to approximate sample distributions of each type of flood-generating category.

Hydroclimatic Classification of Flood Events

Each flood event was classified into one of eight categories representing the major flood-generating mechanisms in the Gila River basin. The hydroclimatic categories were defined after reviewing daily surface and upper air weather maps and maps of the basin-wide precipitation patterns for each flood episode during 1950 to 1980. In order to be as consistent and objective as possible when assigning flood events to a category, a

detailed decision tree or flow chart was designed. Figure 6 shows a greatly simplified version of this chart. Decisions were made at branching points in the chart on the basis of the precipitation pattern, daily synoptic weather maps, and antecedent conditions in the basin. The general pathway through the decision tree consisted of the following.

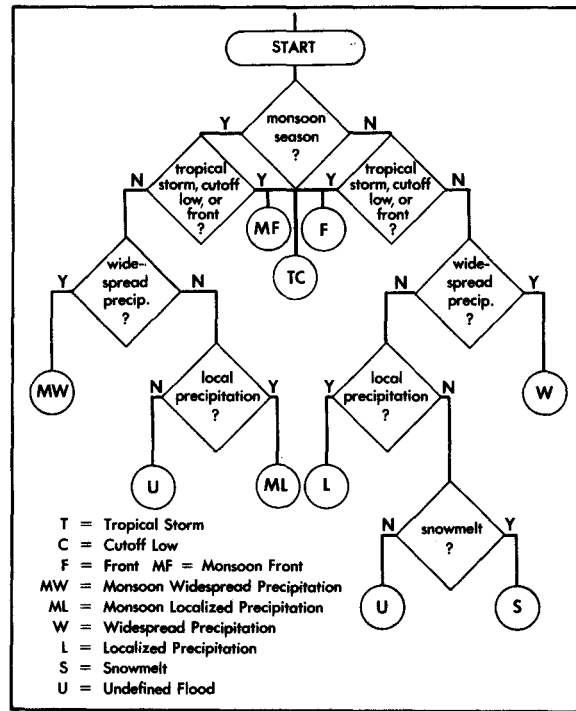


Figure 6. Decision tree for assigning a hydroclimatic classification to each flood event.

After first determining whether or not the humid summer circulation pattern, called the "summer monsoon" (M), had been established, weather maps were examined to see if major synoptic features such as tropical storms (T), cutoff lows (C), or fronts (F) were affecting the precipitation pattern of a developing flood. If no such features were observed, a determination was made of the "widespread synoptic" (W) vs. "local convective" (L) nature of the precipitation over the basin on the basis of daily precipitation totals. In the widespread synoptic situation, although no major synoptic feature appeared on the weather map, precipitation occurred at stations throughout the basin, indicating that some large scale circulation control was in operation introducing moist and unstable air into the area. In the local convective situation, precipitation was recorded at only a few stations in the basin, indicating that

a very localized weather pattern produced the flood. During the non-monsoon season, evidence for snowmelt as a source of flooding was also evaluated. Combinations of classifications were possible, but an attempt was made to determine the dominating flood-generating feature. Because tropical storms and cutoff lows frequently occurred together and are synoptically related, a combined tropical storm/cutoff low class (TC) was formed. If no classification or probable classification could be identified, the flood event was designated as "undefined" (U).

Results of the Classification

Monthly frequencies of floods produced by each hydroclimatic category are shown in Table 1, along with percentages reflecting the relative importance of each type of flood-generating mechanism in the study area. The most important categories for the Gila River basin as a whole are, in descending order, monsoon widespread (MW), frontal (F), monsoon local (ML), and tropical storm/cutoff low (TC). Table 1 also shows the clear dominance of some categories in certain months, such as the tropical storm/cutoff low type in September and October, and the frontal type in December through March.

Table 1 Monthly Frequencies of Floods in Hydroclimatic Categories

	Oct	Nov	Dec	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Total	%
TC	106	33	5	11		7	11	2	7	14	59	100	355	13
F	20	27	175	157	123	129	19	10	1			20	681	24
L	11	1	1		3	10	49	17	8			6	106	4
W	7	2	13	18	37	18	6	5	4			17	127	4
MF									1	24	17	5	47	2
ML									4	186	310	71	571	20
MW									1	274	472	55	802	28
S			1	4	18	60	48						131	5
Total	144	63	195	190	181	224	133	34	26	498	858	274	2820	100

Table 2 reveals the relative importance of each category at selected individual stations in the Gila River basin. It is immediately apparent that different parts of the study area are affected to varying degrees by the eight categories of flood-generating mechanisms. The more northern stations are strongly dominated by frontal floods, while the more southern stations -- including the southeastern and southwestern stations -- are dominated by monsoon widespread floods. Tropical storms and cutoff lows are an important source of flooding at stations throughout the basin, but snowmelt as a flood-generating mechanism is limited to the northern, high elevation catchments that drain the Mogollon Rim.

Table 2 Classification Frequencies for Selected Stations, 1950-1980

	TC	F	L	W	MF	ML	MW	S
Eastern stations:								
1 Gila near Clifton	14	18	2	2	2	24	27	1
2 San Francisco at Clifton	23	28	2	4	1	21	34	5
5 San Carlos near Peridot	16	29	2	7		28	31	3
Southern stations:								
7 San Pedro at Charleston	12	4		1	3	35	48	
12 Santa Cruz at Tucson	18	11	1	2	4	40	64	
Northern stations:								
20 Salt near Roosevelt	15	50	3	6		7	17	12
21 Tonto near Roosevelt	16	58	1	10		11	30	2
22 Oak Creek near Cornville	10	41		9	2	8	16	13
25 Verde above Horseshoe Dam	12	47		7	1	4	13	14
Western stations:								
26 Agua Fria near Mayer	13	20	4	4	2	30	40	
28 Hassayampa near Wickenburg	20	40		8	2	16	31	

HYDROCLIMATICALLY-DEFINED MIXED DISTRIBUTIONS

Theoretically, if each hydroclimatic category reflects a homogeneous climatic process, the subset of floods generated at a given station by a specific hydroclimatic mechanism should represent a homogeneous sample drawn from the population of all possible floods produced by that particular mechanism. Furthermore, the separation of a flood record into each of its hydroclimatic flood-generating components simulates the decomposition of a single-process frequency distribution into a mixed distribution having a physical explanation for each of the component parts.

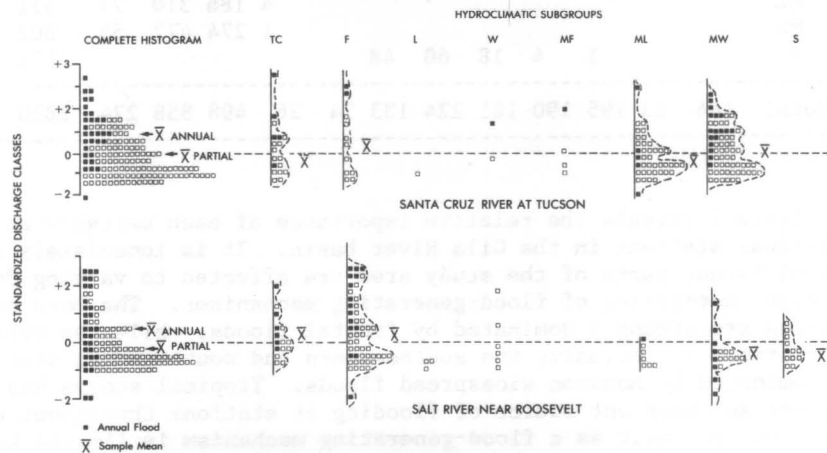


Figure 7. Decomposition of two flood series into hydroclimatic subgroups. Each square represents one flood event.

Histograms of the standardized flood discharges generated by each mechanism serve as an approximation of the decomposition of a flood series into its hydroclimatically-defined component parts. Figure 7 displays this decomposition for two of the Gila basin stations: the Santa Cruz River at Tucson in the southern part of the region with a drainage area of 5755 sq km (Station 12), and the Salt River near Roosevelt in the northern part of the basin with a drainage area of 11,153 sq km (Station 20). The subgroup histograms at each station effectively reveal the nature of the flooding process in these two different parts of the basin. In the southern Santa Cruz drainage, summer monsoon widespread synoptic precipitation (MW) is the major cause of both partial and annual series flooding, and the dominance of this mechanism is reflected in the similarity of shape between the sample frequency distribution of MW floods and the combined histogram of the series. However, some of the largest floods in the record were produced by the tropical storm/cutoff low and frontal precipitation mechanisms. These distributions, (to the degree that they are defined by the sample), are highly positively skewed with large variances. In October 1983, a TC type event associated with Tropical Storm Octave resulted in the largest flood on record for the Santa Cruz River. The inclusion of this event in Figure 7 would skew the TC distribution even more. The monsoon local precipitation mechanism (ML) is also a major generator of floods in the Santa Cruz, but the discharges in this subset tend to be small and few annual floods are produced, despite the high frequency of this type. Snowmelt is not important as a flood-generating mechanism in the Santa Cruz watershed.

The Salt River to the north exhibits a different sensitivity to the eight hydroclimatic categories. The frontal precipitation mechanism generates the most floods, as well as the largest floods. The tropical storm/cutoff low category also contributes some large floods, although unlike the Santa Cruz, the TC mechanism is not responsible for the largest flood on record. Monsoon local precipitation is relatively ineffective as a flood-generator in the large Salt River basin, but the monsoon widespread synoptic precipitation mechanism is responsible for several of the annual floods occurring in summer. Snowmelt produces some flooding in the basin, but discharges are relatively small.

DISCUSSION

The decomposition of a flood series into hydroclimatic subgroups is an effective method of exploring the underlying physical elements of the flooding process and how they combine to produce the flood frequency distribution that is "seen" by a sample of floods recorded at a gaging station. The method is also very useful for identifying which streams will respond in like manner to a given climatic input so that spatially homogeneous stations can be grouped together for regional studies. Due to the occasional occurrence of several atmospheric mechanisms operating together to produce floods, the eight hydroclimatic subgroups may not represent completely homogeneous components of the mixed distribution. In fact, the histograms in Figure 7 suggest that some hydroclimatic categories have multiple-peaked distributions of their own that may

reflect two or more modes of operation within a given flood-generating process. In addition, nonclimatic sources of nonhomogeneity, such as land use changes or channel modifications, may obscure the uniqueness of each hydroclimatic subgroup.

The technique needs to be further refined in order to address the statistical significance of frequency distributions derived from different hydroclimatic mechanisms. Due to the small sample sizes of some of the subgroups, special modeling or simulation will be required to obtain parameter estimates of the hydroclimatic component distributions. The results of this study suggest that the means, variances, and shapes of the flood frequency distributions of different hydroclimatic subgroups may have a physical basis that can be linked to the inherent nature of the flood-generating process. For example, tropical storm/cutoff low floods at many of the stations tended to have highly positively skewed distributions with extremely large variances. This can be physically explained by the erratic nature of eastern North Pacific tropical storm tracks and the rare but extremely large discharges that can occur when a storm, on occasion, does move directly into the Gila River basin.

If reliable parameter estimates of the component distributions can be made, hydroclimatically-defined mixed distributions may be of greatest use in extending frequency curves to predict rare events. In the Santa Cruz basin, tropical storms have been responsible for the largest floods on record; while in the Salt River system, frontal storms produced the highest recorded discharges. A better knowledge of the frequency distribution of floods produced by each of these mechanisms would greatly improve our ability to predict the extreme events occurring in the tails of the composite distribution.

SUMMARY AND CONCLUSIONS

Climate is often proposed as a potential source of mixed distributions in flood series, but the separation of flood records into climatic subgroups is usually based on a simple seasonal division that does not take into consideration the variety of atmospheric circulation mechanisms which generate floods at different times of the year in various parts of a large drainage basin. By classifying events in a flood series on the basis of the climatic mechanism which generates each flood, hydroclimatic subgroups of the series can be defined. Theoretically, these subgroups can be interpreted as samples representing the component frequency distributions of a composite, mixed distribution of floods.

The technique presented here defined mixed distributions in peak flow data for thirty gaging stations in the climatically sensitive Gila River basin for the period 1950-1980. The flood events were hydroclimatically classified into eight main categories using surface and upper air weather maps and precipitation data. The analysis demonstrated that floods in the basin originate from a variety of atmospheric processes and suggested that the parameters of the flood frequency distributions of different hydroclimatic subgroups may have a physical basis.

This study is believed to be the first to attempt to define the mixed components of a distribution by classifying each event in a flood

series according to the synoptic weather patterns responsible for the event. Refinement of the technique and larger sample sizes could result in improved parameter estimates of the component distributions, as well as a better understanding of the physical basis for rare flood events.

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